Lower Bounds for Linear Algebraic Computation

. . .and some more suff

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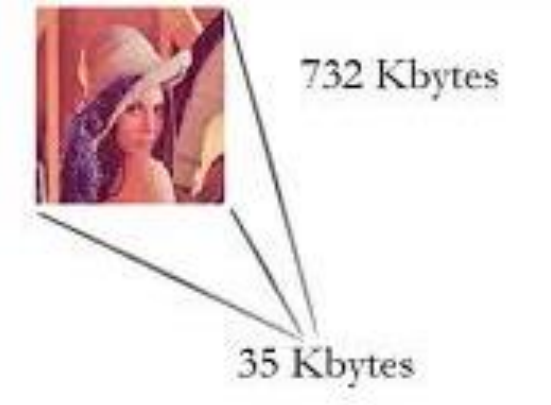
הקרן הלאומית למדע

Fourier Transform Everywhere



Fourier Transform in TCS and Combinatorics

- Fast polynomial multiplication (via FFT)
- Finding correlations in data
- Fast dimensionality reduction
  - (Fast Johnson-Lindenstrauss Transform)
  - [A, Chazelle, Liberty…]
- Property testing
  - Boolean functions
    [Bourgain, Kindler, Dinur, O'Donnell, Mossel, Friedgut…]
  - K-wise independence [Alon, Andoni, Kaufman, Matulef, Rubinfeld, Xie..]
- Compressed sensing
  (via RIP matrix construction)
  [Candès, Tao, Elad, Rauhut, Rudelson, Vershynin, Tropp, Krahmer, Ward..]
- Hardness of approximation (PCP theorem)
  [Arora, Safra, Hastad,…]
- Social choice
  - Quantitative versions of Arrow's impossibility theorem [Friedgut, Kalai,…]

Fourier Transform in Learning Theory

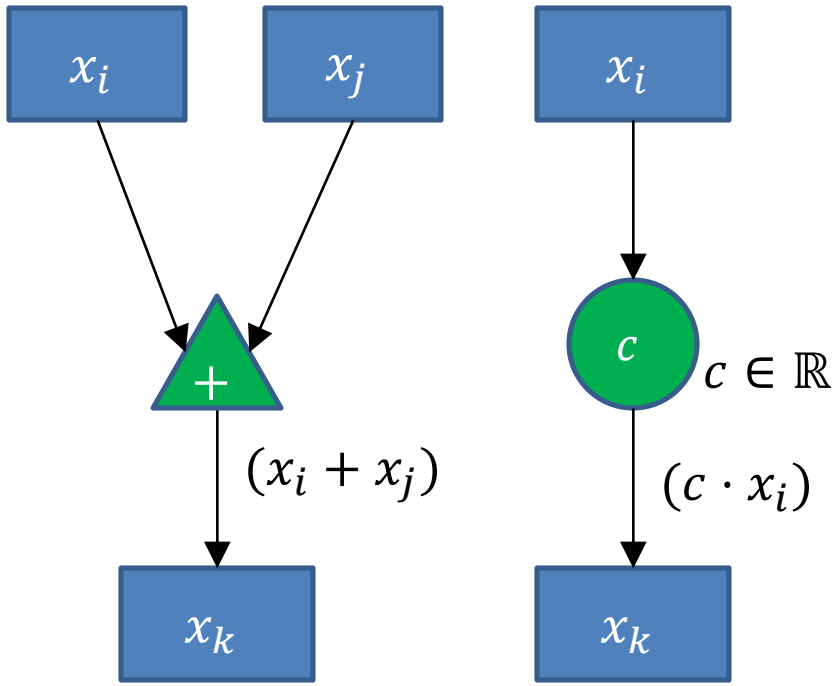
- Learning constant depth circuits
  - [Linial, Mansour, Nisan'89]…
- Learning DNF's
  - [Mansour'95]
  - [Jackson'94]
  - [Klivans+Servedio'01]
  - [Bshouty, Mossel, O'Donnell, Servedio'05]
- Learning Juntas
  - [Mossel, O'Donnell, Servedio'04]
- Learning decision trees
  - [Kushilevitz, Mansour'93]
  - [O'Donnell, Servedio'07]

Research on Fourier Complexity

There is an algorithm FFT that computes Fx given x in time $O(n \log n)$

Very little known about lower bounds

Linear Computational Model



How Many Operations Required for Computing FT of x given x?

- Very few results, very restricted
- [Papadimitriou 1979]
- [Morgenstern 1974]
- [Ailon 2013, 2014, 2015]

[Ailon 2015]

Lower Bound:

$$\Omega\left(\frac{n \log n}{R}\right)$$

My bits are falling!

Condition Number
Higher = Less Accurate

The faster you go, the more information you lose!

(If you speedup up by R you lose Ω(n log R) bits.)

Research Problems

1. **Main Conjecture**: Lower bound should be stronger (visit me to understand why)
2. **Applications**: Fourier used *everywhere* (polynomial mult., integer mult., correlation finding, dimensionality reduction, streaming for large scale linear algebra). How would loss of information affect these applications?
3. **Other problems:** Can the techniques be used for lower bounding other linear functions? (eg Matrix multiplication)

More research in learning theory…